

Pi Pi Scattering and Scalar Mesons in an Effective Chiral Lagrangian^{*)}

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In this talk I summarize recently proposed mechanisms to understand $\pi\pi$ scattering to 1 GeV in an effective chiral Lagrangian. The Lagrangian includes higher resonances in addition to pions consistently with the chiral symmetry. Iso-spin zero S -wave partial wave amplitude is reproduced up till about 1.2 GeV by including a pion self-interaction and resonant pole exchanges of ρ , $f_0(980)$ and σ derived from the effective chiral Lagrangian. The best fit shows that σ has a mass of around 560 MeV and a width of about 370 MeV.

§1. Introduction

QCD is known to be the fundamental theory of the strong interaction. However, it is very difficult to reproduce experimental data directly from QCD. One clue to study low energy properties of QCD is given by the structure of the chiral symmetry, which approximately exists in the QCD Lagrangian and is broken by the strong interaction of QCD. Another clue is given by the $1/N_C$ expansion to QCD.³⁾

The $\pi\pi$ scattering has been studied as an important test of the low energy properties of QCD. The experimental data in the low energy region near $\pi\pi$ threshold can be reproduced by using the information from chiral symmetry. This situation is easily understood by using a chiral Lagrangian which includes pions only. In addition, by including higher derivative terms together with one-loop effects, the applicable energy region is enlarged. This systematic low energy expansion is called the chiral perturbation theory.⁴⁾

In the higher energy region, however, the one-loop amplitude of the chiral perturbation theory violates the unitarity around 400 – 500 MeV in the $I = 0$ S -channel.⁵⁾ For the P -wave amplitude, we have the ρ meson, and the chiral perturbation theory may break down at the resonance position. The explicit inclusion of resonances in the high energy region easily reproduces the amplitude. In such a case, however, it is important to consider clues to QCD other than the chiral symmetry.

In the large N_c limit QCD becomes a theory of weakly interacting mesons, and the $\pi\pi$ scattering is expressed by infinite sum of tree graphs. However, we cannot actually include infinite number of resonances. Moreover, the forms of interactions are

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not fully determined in the large N_C limit. Nevertheless, some encouraging features were previously found in an approach which truncated the particles appearing in the effective Lagrangian to those with masses up to an energy slightly greater than the range of interest.⁶⁾ Then we constructed a resonance model, where we truncated the theory, and included particles with masses up to an energy slightly greater than the range of interest. Moreover, the chiral symmetry was used to restrict forms of interactions, i.e., the effective Lagrangian was constructed by using the information of chiral symmetry. This seems reasonable phenomenologically and is what one usually does in setting up an effective Lagrangian.

In this talk I concentrate on the energy region below 1 GeV. For the established resonances lighter than 1 GeV, ρ and $f_0(980)$ are contained in the particle data group (PDG) list⁷⁾ (see Table I). However, the width of $f_0(980)$ is not well determined.

| | $I^G(J^{PC})$ | $M(\text{MeV})$ | $\Gamma(\text{MeV})$ |
|--------------------|---------------|-----------------|----------------------|
| $\sigma(400-1200)$ | $0^+(0^{++})$ | 400–1200 | 600–1000 |
| $\rho(770)$ | $1^+(1^{--})$ | 769.3 | 150.2 |
| $f_0(980)$ | $0^+(0^{++})$ | 980 | 40–100 |

Table I. Resonances included in the $\pi\pi \rightarrow \pi\pi$ channel as listed in the PDG⁷⁾.

Moreover, the existence of a light scalar σ is suggested by several authors.⁸⁾ Here I will determine these resonance parameters by fitting to the $I = 0$ S -wave $\pi\pi$ scattering amplitude.

This talk is organized as follows. In section 2 I will show the interactions among the higher resonances and two pions, which are derived from an effective chiral Lagrangian. Section 3 is the main part of this talk, where I will show how to regularize the amplitude, and fit the resonance parameters to the experimental data of the $I = 0$, $J = 0$ partial wave amplitude. Finally, a summary is given in section 4.

§2. Resonance Model

In this section I will show the interactions of the higher resonances, listed in Table I, with two pions. These interactions are derived from an effective chiral Lagrangian which includes the higher resonances consistently with the chiral symmetry. The starting effective chiral Lagrangian includes pions through the non-linear realization of the chiral symmetry breaking.

First I include the vector meson as a gauge field of chiral symmetry⁹⁾, which is equivalent to the hidden local gauge method (See, for a review, Ref. 10.) at tree level. This leads to the following $\rho\pi\pi$ interaction:

$$\mathcal{L}_\rho = g_{\rho\pi\pi} \vec{\rho}_\mu \cdot (\partial^\mu \vec{\pi} \times \vec{\pi}) , \quad (2.1)$$

where $g_{\rho\pi\pi}$ is the $\rho\pi\pi$ coupling constant.

Next, I include scalar resonances, σ and $f_0(980)$. These are iso-singlet fields. Inclusion of an iso-singlet scalar field consistently with the chiral symmetry leads to the following interaction among one scalar and two pions:

$$\mathcal{L}_f = -\frac{\gamma_f}{\sqrt{2}} f \partial_\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} \quad (f = \sigma, f_0(980)) . \quad (2.2)$$

Here I should note that the chiral symmetry requires derivative-type interactions between the scalar and pseudoscalar mesons.

§3. Fit to $\pi\pi$ Scattering to 1 GeV

In this section, I will calculate the $I = 0$ S -wave $\pi\pi$ scattering amplitude by including resonances as explained in the previous section.

The most problematic feature involved in comparing the leading $1/N_C$ amplitude with experiment is that it does not satisfy unitarity. Since the mesons have zero width in the large N_C limit, the amplitude diverges at the resonance position. Thus in order to compare the $1/N_C$ amplitude with experiment we need to regularize the resonance contribution. Here let me summarize the regularizations.

Ordinary narrow resonances such as ρ are regularized by including the width in the denominator of the propagator (the Breit-Wigner form):

$$\frac{M\Gamma}{M^2 - s - iM\Gamma} . \quad (3.1)$$

This is only valid for a narrow resonance in a region where the *background* is negligible. Note that the width in the denominator is related to the coupling constant.

For a very broad resonance there is no guarantee that such a form is correct. A suitable form turned out to be of the type:

$$\frac{MG}{M^2 - s - iMG'} , \quad (3.2)$$

where the parameter G' is a free parameter. This G' is not related to the coupling constant.

Even if the resonance is narrow, the effect of the background may be rather important. This seems to be true for the case of $f_0(980)$. Demanding local unitarity in this case yields a partial wave amplitude of the well known form:¹¹⁾

$$\frac{e^{2i\delta}M\Gamma}{M^2 - s - iM\Gamma} + e^{i\delta} \sin \delta , \quad (3.3)$$

where δ is a background phase (assumed to be slowly varying). I will adopt a point of view in which this form is regarded as a kind of regularization of the model. Of course, non zero δ represents a rescattering effect which is of higher order in $1/N_C$. The quantity $e^{2i\delta}$, taking $\delta = \text{constant}$, can be incorporated into the squared coupling constant connecting the resonance to two pions. In this way, crossing symmetry can be preserved. The non-pole background term in Eq. (3.3) and hence δ is to be predicted by the other pieces in the effective chiral Lagrangian.

Another point which must be addressed in comparing the leading $1/N_C$ amplitude with experiment is that it is purely real away from the singularities. The regularizations mentioned above do introduce some imaginary pieces but these are clearly more model dependent. Thus it seems reasonable to compare the real part of the predicted amplitude with the real part of the experimental amplitude.

Let me start from the *current algebra* + ρ contribution. The predicted curve is shown in Fig. 1 of Ref. 1. Although the introduction of ρ dramatically improves unitarity up to about 2 GeV, R_0^0 violates unitarity to a lesser extent starting around 500 MeV. To recover unitarity, we need a negative contribution to the real part above this point, while below this point a positive contribution is preferred by experiment. Such behavior matches with the real part of a typical resonance contribution. The resonance contribution is positive in the energy region below its mass, while it is negative in the energy region above its mass. Then I include a low mass broad scalar resonance, σ . The σ contribution to the real part of the amplitude component $A(s, t, u)$ is given by

$$A_\sigma(s, t, u) = \frac{\gamma_\sigma^2}{2} \frac{(s - 2m_\pi^2)^2}{M_\sigma^2 - s - iM_\sigma G'} , \quad (3.4)$$

where the factor $(s - 2m_\pi^2)^2$ is due to the derivative-type coupling required for chiral symmetry in Eq. (2.2). G' is a parameter which we introduce to regularize the propagator. It can be called a width, but it turns out to be rather large so that, after the ρ and π contributions are taken into account, the partial wave amplitude R_0^0 does not clearly display the characteristic resonant behavior.

A best overall fit is obtained with the parameter choices; $M_\sigma = 559$ MeV, $\gamma_\sigma = 7.8 \text{ GeV}^{-1}$ and $G' = 370$ MeV. The result for the real part R_0^0 due to the inclusion of the σ contribution along with the π and ρ contributions is shown in Fig. 1. It is seen that the unitarity is satisfied and there is a reasonable agreement with the experimental points^{12), 13)} up to about 800 MeV.

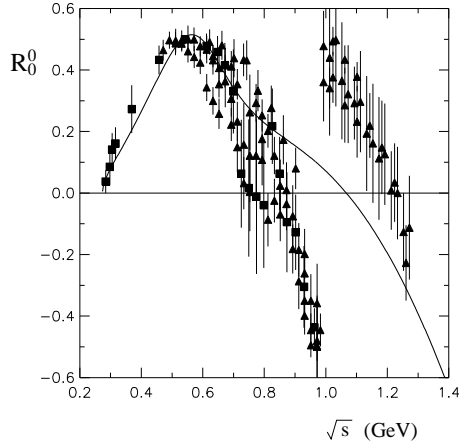


Figure 1: The solid line is the *current algebra* + ρ + σ result for R_0^0 . The experimental points, in this and succeeding figure, are extracted from the phase shifts (\square : Ref. 12, \triangle : Ref. 13.).

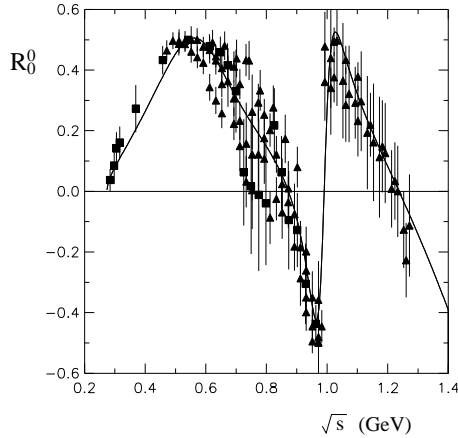


Figure 2: The solid line is the *current algebra* + ρ + σ + $f_0(980)$ result for R_0^0 obtained by assuming the values in Table II for the σ and $f_0(980)$ parameters.

Next, let me consider the 1 GeV region. Reference to Fig. 1 shows that the experimental data for R_0^0 lie considerably lower than the $\pi + \rho + \sigma$ contribution between 0.9 and 1.0 GeV and then quickly reverse sign above this point. This is

caused by the existence of $f_0(980)$. As we can see easily, a naive inclusion of $f_0(980)$ does not reproduce the experimental data, since the real part of the typical resonance form gives a positive contribution in the energy region below its mass, while it gives a negative contribution in the energy region above its mass. However, we need a negative contribution below 1 GeV and a positive contribution above 1 GeV.

As I discussed around Eq. (3.3), the effect of the background is important in this $f_0(980)$ region. In this case the background is given by the $\pi + \rho + \sigma$ contribution. Figure 1 shows that the real part of the background is very small so that the background phase δ in Eq. (3.3) is expected to be roughly 90° . This background effect generates an extra minus sign in front of the $f_0(980)$ contribution, as we can see from Eq. (3.3). Thus $f_0(980)$ gives a negative contribution below the resonance position and gives a positive contribution above it. This is exactly what is needed to bring experiment and theory into agreement up till about 1.2 GeV.

A best fit of our parameters to the experimental data results in the curve shown in Fig. 2. Only three parameters γ_σ , G' and M_σ are essentially free. The others are restricted by experiment. Since the total width of $f_0(980)$ has a large uncertainty (40 – 100 MeV in PDG list), we also fit this. In addition we have considered the precise value of M_{f_0} to be a parameter for fitting purpose. The best fitted values are shown in Table II together with the predicted background phase δ and the χ^2 value. The predicted background phase is seen to be close to 90° , and the low lying sigma has a mass of around 560 MeV and a width of about 370 MeV.

| $M_{f_0(980)}$ | $\Gamma_{f_0(980)}$ | M_σ | G' | γ_σ | δ (deg.) | χ^2 |
|----------------|---------------------|------------|------|-----------------|-----------------|----------|
| 987 | 64.6 | 559 | 370 | 7.8 | 85.2 | 2.0 |

Table II. The best fitted values of the parameter together with the predicted background phase δ and the χ^2 value. The units of $M_{f_0(980)}$, $\Gamma_{f_0(980)}$, M_σ and G' are MeV and that of γ_σ is GeV^{-1} .

§4. Summary

In this talk I showed main mechanisms of the analysis done in Ref. 1: (1) motivated by the large N_C approximation to QCD, we include resonances with masses up to an energy slightly greater than the range of interest, and use the chiral symmetry to restrict the forms of the interactions; (2) the *current algebra* + ρ contribution violates the unitarity around 560 MeV region but it is recovered by including the low mass broad resonance σ ⁶⁾; (3) the $\pi + \rho + \sigma$ contribution gives an important background effect to the $f_0(980)$ contribution, i.e., the sign in front of the $f_0(980)$ contribution is reversed by the background effect. The third mechanism, which leads to a sharp dip in the $I = J = 0$ partial wave contribution to the $\pi\pi$ -scattering cross section, can be identified with the very old *Ramsauer-Townsend* effect¹⁴⁾ which concerned the scattering of 0.7 eV electrons on rare gas atoms. The dip occurs because the background phase of $\pi/2$ causes the phase shift to go through π (rather than $\pi/2$) at the resonance position. (Of course, the cross section is proportional to $\sum_{I,J} (2J+1) \sin^2(\delta_I^J)$.) This simple mechanism seems to be all that is required to understand the main feature of $\pi\pi$ scattering in the 1 GeV region.

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